# Block Compressed Sensing For Feedback Reduction in Relay-Aided Multiuser Full Duplex Networks

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Abstract—Opportunistic user selection is a simple technique that exploits the spatial diversity in multiuser relay-aided networks. Nonetheless, channel state information (CSI) from all users (and cooperating relays) is generally required at a central node in order to make selection decisions. Practically, CSI acquisition generates a great deal of feedback overhead that could result in significant transmission delays. In addition to this, the presence of a full-duplex cooperating relay corrupts the fed back CSI by additive noise and the relay's loop (or self) interference. This could lead to transmission outages if user selection is based on inaccurate feedback information. In this paper, we propose an opportunistic full-duplex feedback algorithm that tackles the above challenges. We cast the problem of joint user signal-tonoise ratio (SNR) and the relay loop interference estimation at the base-station as a block sparse signal recovery problem in compressive sensing (CS). Using existing CS block recovery algorithms, the identity of the strong users is obtained and their corresponding SNRs are estimated. Numerical results show that the proposed technique drastically reduces the feedback overhead and achieves a rate close to that obtained by techniques that require dedicated error-free feedback from all users. Numerical results also show that there is a trade-off between the feedback interference and load, and for short coherence intervals, fullduplex feedback achieves higher throughput when compared to interference-free (half-duplex) feedback.

*Index Terms*—Feedback, scheduling, decode-and-forward, fullduplex relaying, compressive sensing.

### I. INTRODUCTION

The rapid demand for data services in cellular networks calls for efficient techniques that provide high data rate with minimum service delay and broader network coverage [1,2]. Relaying, in which an intermediate node aids the basestation (BS) to transmit packets to a destination node, has been recently introduced as a potential solution to provide high data rates and extend the network coverage [1]. Relay technology has already been adopted in a number of industry standards such as IEEE 802.16j mobile multi-hop relaying [3] and IEEE 802.11s mesh networks [4].

In a relay-aided multiuser network, end users may experience independent time-variant fading channels that result in a poor performance if the destination user undergoes a severe channel degradation. These channel fluctuations, however, can be exploited to improve the network performance. This can be done via opportunistic scheduling by granting the transmission to the user that experiences the best channel conditions [5]– [8]. However, this requires full channel state information (CSI) at the BS, which might not be feasible especially when the network is large. In addition to this, the feedback channel is generally subjected to additive noise, fading, and in the presence of a full duplex (FD) relay, relay loop interference. This loop interference is usually unknown and it is modeled as a random variable [9]. Both of these impairments corrupt the fed back CSI might lead to transmission outages if user selection is based on inaccurate feedback information. In view of these concerns, it is therefore imperative to design robust feedback techniques that exploit multiuser diversity while reducing the amount of required feedback resources.

Prior work on multiuser relaying [10,11] investigated several key performance measures in multiuser relay networks. For instance, the work in [10,12,13] derived the outage probability for multiuser relay networks. Nonetheless, the results in [10, 11] assume perfect CSI at the BS and their analysis is limited to half duplex relaying and do not take into account the effect of the feedback overhead on the network performance.

In this paper <sup>1</sup>, we propose a feedback technique for relayaided networks that permits the BS to obtain the CSI of a few strong users in the presence of noise and relay loop interference in the feedback channels. To reduce the feedback overhead, users with SNR above a threshold feedback their CSI, via the relay, in a full duplex manner. This results in a sparse signal that becomes more structured at the output of the relay. This structure is exploited to pose the problem of feedback recovery as a block sparse recovery problem in CS. Block CS recovery exploits the prior information of the signal block size to better differentiate true signal information from recovery artifacts. This leads to a more robust recovery when compared to conventional CS recovery [14]. We demonstrate the effectiveness of the proposed technique and show that that the proposed technique achieves a comparable rate with lower feedback overhead when compared full feedback techniques.

We organize the paper as follows. In section II, we provide the system description. In section III, we present the proposed

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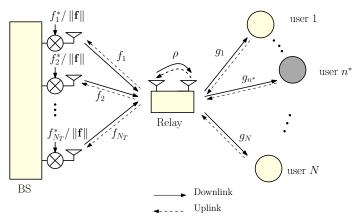


Figure 1: Network Model.

feedback algorithm which we analyze its performance in section IV. Some numerical results are provided in Section V and we some conclusions are drawn in section VI.

### **II. SYSTEM DESCRIPTION**

In this section, we provide a full description of the downlink model as well as the feedback model of the multiuser FD relay-aided network sketched in Figure 1.

## A. Downlink Model

In the downlink, we consider a BS serving N users where the only mean of communication is via a FD relay that operates using the decode and forward (DF) protocol in the downlink. The BS is equipped with  $N_T$  antennas, and the BS-relay channel is modeled as Nakagami- $\theta$  with the following power distribution[15]

$$p_{|f|^2}(x) = \theta^{\theta} \frac{x^{\theta-1}}{\Gamma(\theta)} \exp\left(-\theta x\right), \ x \ge 0, \tag{1}$$

where  $\theta$  is the shape parameter and the scale parameter is given by  $\frac{1}{\theta}$ . The channels between the relay and the users are modeled as Rayleigh fading channels. The Rayleigh assumption arises since mobile users typically suffer severe multipath especially in highly dense and congested areas. Note that this Nakagami-Rayleigh model was also adopted in [10]. The power distribution for the Rayleigh fading is given by

$$p_{|q|^2}(x) = \exp(-x), \ x \ge 0.$$
 (2)

The destination user denoted by  $n^*$  is selected opportunistically using the following rule

$$n^* = \arg\max_n \gamma_n,\tag{3}$$

where  $\gamma_n = \frac{P_r |g_n|^2}{N_0}$  is the instantaneous receive SNR experienced by the *n*th user,  $P_r$  is the relay's total transmit power and  $N_0$  is the noise power. On the relay side, we assume that the loop interference can be modeled as a multiplicative real factor  $\rho \in (0, 1)[16]$  that remains unchanged during the transmission period.

#### B. Uplink Model (Feedback)

Throughout this paper, we assume a reciprocal time-division duplex (TDD) setting such that the downlink channels are the same as its corresponding uplink channels. Moreover, we assume that all channels remain constant during the channel coherence time denoted by  $T_c$  and that the downlink transmission as well as the uplink are performed during the coherence interval. For feedback, we assign a unique signature sequence of dimension M to each user to be used in the feedback period. The sequences, which are known by the BS, are drawn from the columns of a real Gaussian matrix  $\Phi \in \mathbb{R}^{M \times N}$ with i.i.d zero mean entries with normalized variance  $\frac{1}{M}$ . Prior to feedback, all users estimate their channels with the relay via training and only users that experience good channel conditions are allowed to feedback using the following rule

$$x_n = \begin{cases} \gamma_n &, \gamma_n > \gamma_{th}.\\ 0 &, \text{otherwise.} \end{cases}$$
(4)

where  $x_n$  is the feedback SNR of the *n*th user and  $\gamma_{th}$  is a threshold optimized to meet a target scheduling outage probability  $\mathcal{P}_0$  and average sparsity S[11]. We denote by  $\mathbf{x} = [x_1, x_2, \dots, x_N]^t$  the feedback vector and by  $\phi_t = [\phi_{t,1}, \phi_{t,2}, \dots, \phi_{t,N}]$  the *t*th row of  $\boldsymbol{\Phi}$ . Using this notation, the feedback signal by the *n*th user at the *t*th mini-slot is given by  $\phi_{t,n}x_n$ . Let  $y_r(t)$  be the received feedback signal at the *t*th mini-slot. Then,  $y_r(t)$  is given by

$$y_r(1) = \phi_1 \mathbf{x} + z_r(1).$$
 (5)

$$y_r(t) = \phi_t \mathbf{x} + \rho y_r(t-1) + z_r(t), t = 2, ..., M,$$
 (6)

where  $z_r(t)$  is the zero mean additive noise at the relay at the *t*th mini-slot with power  $N_0$ . Since  $\rho \in (0, 1)$ , we assume for tractability that  $\rho$  is small enough such that high powers of  $\rho$  can be neglected, i.e.,  $\rho^k \simeq 0$ , for all  $k \ge K$ , where K is a parameter controlling the accuracy of the approximation. Intuitively high values of K better describes the actual interference at the relay, however it results in more complexity as will be shown in the course of the paper. Consequently, the received signal in (5) can be truncated as follows

$$y_r(t) = \sum_{k=0}^{\min(K-1,t-1)} \rho^k \phi_{t-k} \mathbf{x} + \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k),$$
(7)

where the term  $\kappa = \lceil \frac{K}{2} \rceil$  comes from the fact that  $\mathbf{E} \left[ \left| \rho^k z_r \right|^2 \right]$ , for all  $k \geq \kappa$ . This signal is then forwarded as is to the BS which receives the signal at each of its  $N_T$  antennas. After multiplying by  $\frac{f_i^*}{\|\mathbf{f}\|}$ , the *i*th antenna receives at the *t*th time slot

$$y_{s,i}(t) = \frac{|f_i|^2}{\|\mathbf{f}\|} \sum_{k=0}^{\min(K-1,t-1)} \rho^k \phi_{t-k} \mathbf{x} + \frac{|f_i|^2}{\|\mathbf{f}\|} \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k) + \frac{f_i^*}{\|\mathbf{f}\|} w_i(t),$$
(8)

where  $w_i(t)$  is the zero mean additive noise at the *i*th antenna at the *t*th time slot with power  $N_0$  and  $\mathbf{f} = [f_1, f_2, \cdots, f_{N_T}]^t$ denotes the BS channel vector with the relay. Now, combining the received signals from all the  $N_T$  antennas and normalizing by  $\|\mathbf{f}\|$ , we have

$$y_{s}(t) = \sum_{k=0}^{\min(K-1,t-1)} \rho^{k} \phi_{t-k} \mathbf{x} + \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^{k} z_{r}(t-k) + \sum_{i=1}^{N_{T}} \frac{f_{i}^{*}}{\|\mathbf{f}\|^{2}} w_{i}(t) = \sum_{k=0}^{\min(K-1,t-1)} \rho^{k} \phi_{t-k} \mathbf{x} + \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^{k} z_{r}(t-k) + \sum_{i=1}^{N_{T}} \frac{f_{i}^{*}}{\|\mathbf{f}\|^{2}} w_{i}(t).$$
(9)

The objective of the following section is to exploit the structure in (9) to jointly estimate the feedback signal as well as the loop interference at the relay.

## III. JOINT FEEDBACK AND LOOP INTERFERENCE ESTIMATION

In this section, we introduce the proposed CS based recovery feedback technique and demonstrate how the structure in the feedback signal can be used to estimate the loop interference. The proposed techniquue is composed of two parts. The first part is the identity estimation phase, where we use CS to determine the identity (ID) of the strong users. The second part makes use of the output of the first part to estimate the SNR of the strong users as well as the loop interference.

## A. User ID Estimation Phase

In order to schedule the strongest user, the BS needs to estimate his identity. To achieve that, we exploit the structure in (9) to cast the problem as a block CS recovery. To show that, we define the following entities

$$\phi_{c,n}^{j} \triangleq \left[ \mathbf{0}_{1 \times j}, \phi_{c,n} \left( 1 : M - j \right)^{t} \right]^{t}.$$
$$\mathbf{\Psi}_{(n)} \triangleq \left[ \phi_{c,n}^{0}, \phi_{c,n}^{1}, ..., \phi_{c,n}^{K-1} \right], n = 1, 2, ..., N,$$

where  $\mathbf{0}_{m \times p}$  is the null matrix of dimension  $m \times p$ ,  $\phi_{c,n}$  denotes the *n*th column of  $\mathbf{\Phi}$ . Then, the received signal in (9) can be rewritten in a matrix form as

$$\mathbf{y}_{\mathbf{s}} = \mathbf{\Psi} \boldsymbol{\chi} + \mathbf{z},\tag{10}$$

where  $\Psi = [\Psi_{(1)}, \Psi_{(2)}, ..., \Psi_{(N)}], \quad \chi = \mathbf{x} \otimes [1, \rho, \cdots, \rho^{K-1}]^t, \otimes \text{denotes the Kronecker product}$ and  $\mathbf{z} = [z_1, \cdots, z_M]^t$ , where

$$z_t = \sum_{k=0}^{\min(\kappa-1,t-1)} \rho^k z_r(t-k) + \sum_{i=1}^{N_T} h_i w_i(t),$$
  
$$t = 1, \cdots, M,$$

with  $h_i = \frac{f_i^*}{\|\mathbf{f}\|^2}$ ,  $i = 1, \dots, N_T$ . Given that **x** is sparse with sparsity S, it follows that  $\chi$  is block sparse with sparsity S

and block size K. As a matter of fact, one can use the results in [14] for block sparse recovery to reliably recover  $\chi$ . In fact, it has been shown in [14] that it is possible to reliably recover  $\chi$  with only  $\mathcal{O}\left(KS + S \log \frac{KN}{S}\right)$  measurements. At the end of this phase, an estimate of the support of  $\chi$  is now available which allows us to perform the SNR estimation. This constitutes the subject of the next subsection.

#### B. Joint SNR and Loop Interference Estimation

After the previous part, the BS have an estimate of the support of  $\chi$  denoted by  $\mathcal{J}$ , where  $|\mathcal{J}| = KS$ . This allows to rewrite the linear system in (10) as

$$\mathbf{y}_s = \mathbf{\Psi}_{\mathcal{J}} \chi_{\mathcal{J}} + \mathbf{z},\tag{11}$$

where, we only need to estimate a  $|\mathcal{J}|$  – dimensional vector  $\chi_{\mathcal{J}}$ . This can be done using existing linear estimation techniques such as least squares. However, since it is possible to obtain the covariance matrix of the noise vector z denoted by  $\Sigma_{z}$  ( $\rho$ ) calculated in [17], the best linear unbiased estimator (BLUE) is the appropriate choice in this case. Indeed, we estimate  $\chi_{\mathcal{J}}$  as follows

$$\widehat{\chi}_{\mathcal{J}} = \left( \Psi_{\mathcal{J}}^{t} \Sigma_{\mathbf{z}}^{-1} \left( \rho \right) \Psi_{\mathcal{J}} \right)^{-1} \Psi_{\mathcal{J}}^{t} \Sigma_{\mathbf{z}}^{-1} \left( \rho \right) \mathbf{y}_{s}$$

$$= \chi_{\mathcal{J}} + \epsilon,$$
(12)

where  $\epsilon$  is the estimation error vector after applying the BLUE. However as shown in (12) the estimate  $\hat{\chi}_{\mathcal{J}}$  depends on the noise covariance matrix  $\Sigma_{\mathbf{z}}(\rho)$  which itself depends on  $\rho$ . Thus, as a first step we estimate  $\chi_{\mathcal{J}}$  using least squares (LS) which does not require any prior statistics on the noise.

Also, notice that  $\hat{\chi}_{\mathcal{J}}$  has the same structure as  $\chi_{\mathcal{J}}$ . More precisely,

$$\widehat{\chi}_{\mathcal{J}} = \widehat{\mathbf{x}}_{\mathcal{S}} \otimes \left[1, \rho, \cdots, \rho^{K-1}\right]^t.$$
(13)

Using the relation in (13), it is easy to notice that the first entry of  $\hat{\mathbf{x}}_{S}$  is interference free, thus we can estimate it and use it to estimate  $\rho$  subsequently.

Therefore,  $\rho$  can be estimated as follows

$$\widehat{\rho} = \left[\frac{\widehat{\chi}_{\mathcal{J}}\left((i-1)K+j+1\right)}{\widehat{\mathbf{x}}_{\mathcal{S}}\left(i\right)}\right]^{1/j}, \ j = 1, \cdots, K-1,$$
$$i = 1, \cdots, S.$$

which means that we have (K - 1) S estimate of  $\rho$ . Averaging over all the estimates, we have the following estimate of  $\rho$ 

$$\widehat{\rho} = \frac{1}{(K-1)S} \sum_{i=1}^{S} \sum_{j=1}^{K-1} \left[ \frac{\widehat{\chi}_{\mathcal{J}} \left( (i-1)K + j + 1 \right)}{\widehat{\mathbf{x}}_{\mathcal{S}} \left( i \right)} \right]^{1/j}.$$
 (14)

From (14), we have an estimate of  $\rho$  which we can use to update the noise covariance matrix  $\Sigma_{\mathbf{z}}(\rho)$  by replacing  $\rho$  with  $\hat{\rho}$  and thus refine the estimate of  $\chi_{\mathcal{J}}$  using the BLUE in (12) and based on the structure of  $\Sigma_{\mathbf{z}}(\rho)$  in Lemma 1.

### C. BLUE Error Analysis

In this subsection, we provide an asymptotic equivalent of the noise power at the output of the BLUE in (12). For that, denote by  $\mathbf{R}_{\epsilon}$ , the covariance matrix of  $\epsilon$  in (12), then by basic manipulations, we can show that

$$\mathbf{R}_{\epsilon} = \left(\boldsymbol{\Psi}_{\mathcal{J}}^{t} \boldsymbol{\Sigma}_{\mathbf{z}}^{-1}\left(\boldsymbol{\rho}\right) \boldsymbol{\Psi}_{\mathcal{J}}\right)^{-1}.$$
 (15)

In the following, we focus our attention to study the behavior of the quantity  $\sigma_{\epsilon}^2 = \frac{1}{M} \text{tr} [\mathbf{R}_{\epsilon}]$  which can be seen as the effective noise power at the output of the BLUE. This constitutes the subject of Lemma 2 where, we provide an asymptotic equivalent for  $\sigma_{\epsilon}^2$  when M is large.

Lemma 1: Let S, K and  $\rho$  be fixed and finite . Then as  $M \to \infty$ ,

$$\mathbf{R}_{\epsilon} - \frac{M\mathbf{I}_{KS}}{\operatorname{tr}\left[\boldsymbol{\Sigma}_{\mathbf{z}}^{-1}\left(\boldsymbol{\rho}\right)\right]} \xrightarrow[M \to +\infty]{a.s.} 0.$$
(16)

**Proof:** The result directly follows from the trace lemma in [18, Theorem 3.4].

Thus, based on the previous asymptotic result, we can approximate the noise variance  $\sigma_{\epsilon}^2$  as follows<sup>2</sup>

$$\sigma_{\epsilon}^2 \simeq \frac{M}{\operatorname{tr}\left[\boldsymbol{\Sigma}_{\mathbf{z}}^{-1}\left(\hat{\rho}\right)\right]}.$$
(17)

## D. SNR Back-off

To minimize the impact of the noise on the estimated SNR, we propose to apply a *back-off* strategy as in [11] such that the probability that the estimated SNR is higher than the actual one is minimized. Without loss of generality, let  $\gamma$  and  $\hat{\gamma}$  respectively denote the actual and the estimated SNRs. Then,  $\hat{\gamma} = \gamma + \epsilon$ , where  $\epsilon$  is a Gaussian error (since it results from a linear transformation of a Gaussian noise) with power  $\sigma_{\epsilon}^2$ . Since the noise is Gaussian, it can take any real value, thus an estimated SNR can be higher than the actual one resulting in a system outage. To deal with that, we propose to back-off on the estimated SNR, i.e., substract a constant  $\Delta$  from  $\hat{\gamma}$ . Then, we have

$$\widehat{\gamma} = \gamma + \epsilon - \Delta, \tag{18}$$

where  $\Delta$  is a constant. Then, for a given  $\Delta$ , the back-off efficiency defined as the probability that the estimated SNR is less than the actual one can be expressed as

$$\mathbb{P}\left(\widehat{\gamma} \leq \gamma\right) = \mathbb{P}\left(\epsilon \leq \Delta\right)$$
$$= 1 - Q\left(\frac{\Delta}{\sigma_{\epsilon}}\right).$$
(19)

The optimal value of  $\Delta$  is discussed in the next section.

#### **IV. PERFORMANCE ANALYSIS**

In this section, we analyze the performance of the proposed feedback technique in terms of the achievable throughput. We define the achievable throughput as the number of transmitted *data* bits per unit time (bps/Hz). The achievable throughput  $\mathcal{R}$  can be defined as follows

$$\mathcal{R} = \mathcal{C}. \underbrace{\frac{T_c - MT_{ms}}{T_c}}_{\text{Effective Transmission}}$$
(20)
$$= \mathcal{C} \left(1 - M\tau\right),$$

where C is the transmission rate,  $T_{ms}$  is the duration time needed to communicate a feedback information, i.e.,  $MT_{ms}$  is the total feedback load in seconds and  $\tau = \frac{T_{ms}}{T_c}$  is the fraction of time needed to communicate one feedback information. As shown in equation (20), the throughput combines the effect of both the rate and the feedback load in the overall performance. We start by expressing the rate as

$$\mathcal{C}\left(\Delta\right) = \log\left(1 + \gamma_{e2e} - \Delta\right)\left(1 - \mathcal{P}_{0}\right)\left(1 - Q\left(\frac{\Delta}{\sigma_{\epsilon}}\right)\right),\tag{21}$$

where  $\Delta$  is a back-off factor,  $\gamma_{e2e} = \min\left(\frac{N_T^- \|\mathbf{r}\|^2}{P_r \rho^2 + N_0}, \gamma_{n^*}\right)$ is the end-to-end SNR of the strongest user. The optimal  $\Delta$  denoted by  $\Delta^*$  is chosen in order to maximize the rate in (21). Thus,  $\Delta^*$  satisfies the following equation [11]

$$\left(\frac{1+\gamma_{e2e}-\Delta^*}{\sqrt{2\pi}\sigma_{\epsilon}}\right)\exp\left(-\frac{(\Delta^*)^2}{2\sigma_{\epsilon}^2}\right)\log\left(1+\gamma_{e2e}-\Delta^*\right) + Q\left(\frac{\Delta^*}{\sigma_{\epsilon}}\right) = 1.$$
(22)

Then,  $\Delta^*$  is plugged in the rate formula in (21) to obtain  $\mathcal{C}(\Delta^*)$  which is the maximum rate we can get by applying the back-off strategy.

We now turn our attention to the feedback load which can be expressed as

$$M = \beta \left( KS + S \log \frac{KN}{S} \right), \tag{23}$$

which is nothing but the number of CS measurement needed at the BS to have efficient feedback recovery,  $\beta$  is a constant.

For the simulations, we tabulate the parameters in Table I and plot the ergodic throughput denoted by  $\overline{\mathcal{R}}$  which is the throughput averaged over all channel realizations. The ergodic throughput can thus be expressed as

$$\overline{\mathcal{R}} = \overline{\mathcal{C}} \left( 1 - M\tau \right), \tag{24}$$

where  $\overline{C}$  is the ergodic rate, which has the following expression

$$\overline{\mathcal{C}} = \mathbf{E} \log \left( 1 + \gamma_{e2e} - \Delta^* \right) \left( 1 - \mathcal{P}_0 \right) \left( 1 - Q \left( \frac{\Delta^*}{\sigma_{\epsilon}} \right) \right),$$
(25)

where the expectation is taken over all channel realizations  $\{f_i\}_{i=1,\dots,N_T}, \{g_n\}_{n=1,\dots,N}$ .

## V. NUMERICAL RESULTS

We compare the proposed feedback technique with two benchmark techniques: 1) its half-duplex version<sup>3</sup> where the

<sup>&</sup>lt;sup>2</sup>The exact value has been derived in closed form in [19] when  $\Sigma_{\mathbf{z}}(\hat{\rho})$  has distinct eigenvalues.

<sup>&</sup>lt;sup>3</sup>Only the uplink is HD in this case and the downlink is FD.

Parameter	Value	Parameter	Value
$P_s$	40 dBm	β	2
$P_r$	15 dBm	$N_0$	1
$N_T$	8 antennas	$\mathcal{P}_0$	0.01
θ	2	K	$\{3, 4, 5\}$

Table I: Simulation parameters

relay forwards the feedback in orthogonal channels, thus twice the number of measurements is needed in this case and no loop interference is present ( $\rho = 0$ ), 2) the full feedback technique which needs all the users to feedback in noiseless dedicated channels, thus the feedback load is linear in N and the rate is maximized. We consider two cases:

i) Small coherence time: In this case, we choose  $\tau = 1/200$ and plot the ergodic throughput in Figure 2. As shown in the figure, the full feedback technique has the worst performance since it has the largest feedback load which means in this situation that the system spends all the resources in feedback rather than actual transmission. The proposed technique beats the half-duplex (HD) version for all K since it consumes less feedback load.

ii) Large coherence time: In this case, we choose  $\tau = 1/2000$ . As shown in Figure 3, for small number of users the full feedback technique achieves the best performance since it has the highest rate and its feedback load is relatively small since we assume large coherence time. The situation is different for large number of users, where the performance of the full feedback technique deteriorates as the feedback load linearly grows with N. However, the HD version gives the best performance despite the fact it has higher feedback load as compared to the proposed technique. This can be explained by the fact that the HD version has a better rate and that for large coherence time, additional feedback load has little impact on the overall performance. This explanation similarly applies to the fact that increasing K gives better performance.

### VI. CONCLUSION

In this paper, we proposed a CS-based feedback strategy for user selection in multiuser full duplex relay networks. Based on the theory of compressed sensing, we were able to cast the problem as a block sparse signal recovery and jointly estimate the feedback signal and the loop interference induced by the simultaneous transmission and reception at the relay. The main conclusion is that for practical scenarios where the coherence time is small, the proposed FD technique performs better than both the HD feedback and the full feedback technique.

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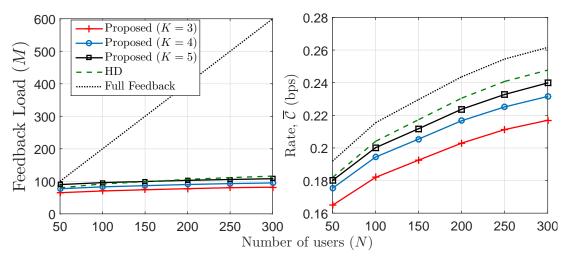


Figure 2: Ergodic rate and Feedback load as a function of the number of users N.



Figure 3: Achievable throughput as a function of the number of users N.